Optimization Techniques For Open Pit Mine Scheduling

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ABSTRACT The economic viability of the modern day mine is highly dependent upon careful planning and management. Declining trends in average ore grades, increasing mining costs and environmental considerations will ensure that this situation will remain in the foreseeable future. The operation and management of a large open pit mine having a life of several years is an enormous and complex task. Though a number of optimization techniques have been successfully applied to resolve some important problems, the problem of determining an optimal production schedule over the life of the deposit is still very much unresolved. In this paper we will discuss some of the techniques that are being used in the mining industry for production scheduling indicating their limitations. In addition, we present a mixed integer linear programming model for the scheduling problems along with a Branch and Cut solution strategy. Computational results for practical sized problems are discussed.

1. INTRODUCTION

The operation and management of a large open pit mine is an enormous and complex task, particularly for mines having a life of many years. Optimization techniques can be successfully applied to resolve a number of important problems that arise in the planning and management of a mine. These applications include: ore-body modelling and ore reserve estimation; the design of optimum pits; the determination of optimal production schedules; the determination of optimal operating layouts; the determination of optimal blends; the determination of equipment maintenance and replacement policies; and many more (Caccetta and Giannini [1986,1990]).

A fundamental problem in mine planning is that of determining the optimum ultimate pit limit of a mine. The optimum ultimate pit of a mine is defined to be that contour which is the result of extracting the volume of material which provides the total maximum profit whilst satisfying the operational requirement of safe wall slopes. The ultimate pit limit gives the shape of the mine at the end of its life. Usually this contour is smoothed to produce the final pit outline.
Optimum pit design plays a major role in all stages of the life of an open pit: at the *feasibility study stage* when there is a need to produce a whole-of-life pit design; at the *operating phase* when pits need to be developed to respond to changes in metal prices, costs, ore reserves, and wall slopes; and towards the *end of a mine's life* where the final pit design may allow the economic termination of a project.

The ultimate pit limit problem has been efficiently solved using the Lerchs-Grossmann [1965] graph theoretic algorithm or Picard's [1976] network flow method (see also Caccetta and Giannini [1986]). A comparative analysis of the two methods is given by Caccetta et al [1994]. Optimum pit design plays an important role in mine scheduling.

The open pit mine production scheduling problem can be defined as specifying the sequence in which "blocks" should be removed from the mine in order to maximise the total discounted profit from the mine subject to a variety of physical and economic constraints. Typically, the constraints relate to: the mining extraction sequence; mining, milling and refining capacities; grades of mill feed and concentrates; and various operational requirements such as minimum pit bottom width.

The scheduling problem can be formulated as a mixed integer linear program (MILP). However, in real applications this formulation is too large, in terms of both the number of variables and the number of constraints, to solve by any available MILP software.

2. MODEL

In this section we outline some of the methods that have been proposed for various mine development problems. We begin with the basic block model of an ore body, then we present a mixed integer linear programming formulation of the scheduling problem.

2.1 Block Model

Though a number of models are available, the regular 3D fixed-block model is the most commonly used and is the best suited to the application of computerized optimization techniques (Gignac [1975] and Kim [1979]). This model is based on the ore body being divided into fixed-size blocks. The block dimensions are dependent on the physical characteristics of the mine, such as pit slopes, dip of deposit and grade variability as well as the equipment used. The centre of each block is assigned, based on drill hole data and a numerical technique, a grade representation of the whole block. The numerical technique used is some grade extension method such as: distance weighted interpolations, repression analysis, weighted moving averages and kriging (Gignac [1975]).

Using the financial and metallurgical data the net profit of each block is determined. The wall slope requirements for each block are described by a set (typically 4 to 8) of azimuth-dip pairs. From these we can identify for each block x the set of blocks S_x which must be removed before block x can be mined. This collection of blocks, $x \cup S_x$, is usually referred to as a "cone".

The key assumptions in the block model are: the cost of mining each block does not depend on the sequence of mining; and the desired wall slopes and pit shape can be approximated by the removed blocks.

2.2 A MILP Formulation of The Scheduling Problem

Let

- T is the number of periods over which the mine is being scheduled.
- N is the total number of blocks in the ore body.
- c_i^t is the profit (in NPV sense) resulting from the mining of block i in period t.
- O is the set of ore blocks.
- W is the set of waste blocks.
- t_i is the tonnage of block i.
- m^t is the tonnage of ore milled in period t.
- S_i the set of blocks that must be removed prior to the mining of block i.
- $x_i^t = \begin{cases} 1, & \text{if block i is mined in periods 1 to t} \\ 0, & \text{otherwise.} \end{cases}$
- ℓ_0^t lower bound on the amount of ore that is milled in period t.
- u_0^t upper bound on the amount of ore that is milled in period t.
- u_w^t upper bound on the amount of waste that is mined in period t.

Then the MILP formulation is:

Maximize

$$Z = \sum_{t=2}^{T} \sum_{i=1}^{N} (c_i^{t-1} - c_i^t) x_i^{t-1} + \sum_{i=1}^{N} c_i^T x_i^T \tag{1}$$

subject to

$$\sum_{i \in O} t_i x_i^1 = m^1 \tag{2}$$

$$\sum_{i \in O} t_i (x_i^t - x_i^{t-1}) = m^t, \quad t=2,3,\dots,T. \tag{3}$$

$$\sum_{i \in W} t_i x_i^1 \le u_w^1 \tag{4}$$

$$\sum_{i \in W} t_i (x_i^t - x_i^{t-1}) \le u_w^t, \quad t = 2,3,\dots,T. \tag{5}$$

$$x_i^{t-1} \le x_i^t, \quad t = 2,3,\dots,T. \tag{6}$$

$$x_i^t \le x_j^t, \quad t=1,2,\dots,T, j \in S_i; \quad i = 1,2,\dots,N. \tag{7}$$

$$\ell_0^t \le m^t \le u_0^t, \quad t = 1,2,\dots,T. \tag{8}$$

$$x_i^t = 0,1, \quad \text{for all i, t.} \tag{9}$$

Constraints (2), (3) and (8) ensure that the milling capacities hold. Constraints (4) and (5) ensure that the tonnage of waste removed does not exceed the prescribed upper bounds. Constraints (6) ensures that a block is removed in one period only. Constraints (7) are the wall slope restrictions. This formulation can be extended to include other factors such as: different ore types; maximum vertical depth; minimum pit bottom width; and stockpiles.

The above formulation has NT 0-1 variables, and (N+2)T + N(d-1) linear constraints, where d is the average number of elements in a cone. Typically

T is around 10, N is 100,000 for a small pit and over 1,000,000 for a larger pit. Consequently the MILP's that arise are much too large for direct application of commercial packages. However, as we demonstrate in this paper, the structure of the problem can be exploited to develop computational strategies that produce provably good solutions.

3. SOLUTION METHODS

Solving MILP's such as (1) – (9) is a difficult and challenging task. Indeed, in the mining context, the lack of an immediate optimization technique has led the mining industry to focus on easy subproblems. The usual approach is to first determine the final pit outline and then through a series of refinements mining schedules are generated. The final pit outline is determined by smoothing the contour produced by solving the ultimate pit limit problem; that is, the solution to the problem (1) subject to (7) with T = 1 and (9).

3.1 Parameterization Method

In their paper Lerchs and Grossmann [1965] introduced the concept of parametric analysis in order to generate an extraction sequence. They considered the undiscounted model and varied the economic value of each block i from c_i to $(c_i - \lambda)$ for varying $\lambda \geq 0$. An increasing sequence of λ values gives rise to a set of nested pits. These pits can be used to produce a production schedule. Since this early work a number of authors have considered the implementation aspects of this method and its variations (Francois-Bongarcon and Guibal [1984], Caccetta et al. [1998a, 1998b], Caléou [1988], Dagdelen and Johnson [1986], Matheron [1975a, 1975b], and Whittle [1993, 1998]).

The mostly widely used scheduling software package that is based on parameterization, is Whittle's Four-D and Four-X [1993, 1998]; the latter allows for multiple ore types in the calculation of block costs. Whittle suggests that a "best" mining schedule comes from extracting each of the nested pits in turn and a "worst" schedule comes from extracting the ore bench by bench.

The main advantages of the Whittle approach include

- the nested pits can be determined efficiently as each requires the solution of an ultimate pit limit problem.
- the identification of clusters of high grade ore in the model.
- a measure in the design of the final pit contour subject to a change in price and thus some sensitivity analysis can be performed.

The main disadvantages include:

- time and other variable factors (for example, extraction rate, different ore types, blending, etc.) are only implicitly included in the optimization through modifying the cost function.
- the possibility of a large increment in the size of the pit from one nested pit to the next. This is referred to as the "gapping problem" and it arises because there is no clear method for choosing the values of λ.
- optimality is not guaranteed. Indeed the "best" schedule may not even provide an upper bound for the NPV of the mine.

Another commercially available package which extends the nested pit approach is the Earthworks NPV Scheduler [www.earthworks.com.au]. This package first generates the nested pits and then using these, the pushbacks are defined heuristically. The criteria for the pushbacks is to keep them as close as possible to the extraction sequence suggested by the nested pits taking into consideration equipment access. Finally, a restricted tree search procedure is used to resequence the pushback removal to increase the NPV. A major advantage of this package is that it may produce schedules that are more likely to be acceptable to mining engineers because practical spacial constraints are taken into account when defining the pushbacks.

Unfortunately, all methods that use the above nested pits approach in a sequential optimization procedure may produce a schedule that varies considerably from the optimum. Indeed, even a feasible solution cannot be guaranteed.

3.2 MILP Approach :

The major computational difficulty with MILP formulations has been the size of the problem. Below we outline two approaches for solving the MILP formulations.

Recently, Combinatorics Pty Ltd (Western Australia) has released the package MineMax for long term mine scheduling. The MILP is solved using a commercial package (for example, CPLEX). Our understanding is that if the problem is too large for the MILP solver, or if a solution is not obtained within a prescribed time period, then a second option is offered. This option is to solve each MILP formulation for free variables on a period by period basis.

The main disadvantage of MineMax is that it is capable of solving only very small size problems due to the large number of integer variables and constraints.

Caccetta et al [1998] proposed a Lagrangian relaxation method for solving the MILP. At each step a problem similar to the ultimate pit limit problem is solved optimally with additional constraints dualized. Subgradient optimization is used to reduce the duality gaps. The method is tested on a real ore body with 20,979 blocks and 6 time periods. The schedules obtained are within 5% of the theoretical optimum. The main problem with the method is resolving the duality gaps. However, the subproblems are useful in producing solutions using a heuristic. In fact, the heuristic solution obtained for the real ore body is within 2% of the theoretical optimum.

3.3 Heuristics

Runge Mining Pty Ltd have developed the XPAC Autoscheduler package (www.runge.com/xpac) for mine scheduling. Their heuristic approach is based on the method proposed by Gershon [1987] which iteratively selects blocks to be extracted on a period by period basis. A weighted function is used to determine the removal sequence. At each step only blocks whose predecessors have been mined are considered. The advantage of the method is its speed. Its main use is an interactive tool where the user can see a large number of scenarios by fixing in and out blocks and running the heuristic. The main disadvantages are: the search is myopic; no guarantee of finding a feasible solution; the obtained solution may be far from optimal. The method has been applied to models with up to 100,000 blocks.

Tolwinski and Underwood [1996] proposed a method which combines concepts from stochastic optimization and artificial neural networks with heuristics exploiting the structure of the mine. The method works by modelling the development of the mine as a sequence of pits (states) where each pit differs from the previous pit by the removal of one block (state change). A probability distribution based on the frequency with which particular states occur is used to determine the state changes. Heuristic rules are incorporated to learn these characteristics of the sequence of pits which produce a good, or poor, result. Only small problems can be attempted.

3.4 A New Branch And Cut Method

Our work is motivated by the recent success of this approach to various large combinatorial optimization problems (Caccetta and Hill [1999]). Essentially, the method adds constraints (cuts) at each node within a Branch and Bound procedure. The MILP is relaxed to a linear program (LP) which is solved optimally. At each node, the set K of valid inequalities for the original problem is checked for violations using the relaxed solution. If none are found, the process terminates, otherwise the violating inequalities are added to the relaxed problem and the process is repeated. If the process terminates in a non optimal solution, it is necessary to branch. We now detail some of the important features of our method which exploits the structure of the problem.

Key Features:

1. The block model is reduced to only include blocks inside the final pit design developed from the ultimate pit. Further reductions are made through consideration of (2) - (5) and (7).

2. The MILP has strong branching variables due to the dependencies between variables ((6) and (7)). Note that setting a variable to 0 or 1 will fix a potentially large number of other variables. Consequently the subsequent LP relaxations are significantly smaller in size. This motivates more branching compared to typical Branch and Cut methods.

3. Cutting planes involving Knapsack constraints are identified using the capacity upper bounds ((2) - (5)) and the block removal dependencies (7). Also cuts are identified through material removal dependencies between benches.

4. Our search strategy involves a combination of best first search and depth first search. The motivation for this is to achieve a "good spread" of possible pit schedules (best first search) whilst benefiting from using depth first search where successive LP's are closely related from one child node to the next. For large problems this often results in provably good solutions being found earlier than a search method geared to establishing an optimal solution.

5. Good lower bounds are generated through the use of an LP-Heuristic. The method works by considering each period in turn and fixing in and out sets of free variables. Cutting planes are then generated for the period, further LP's are solved and further fixing occurs. Throughout the fixing of variables feasibility checks are used. If the heuristic succeeds, or fails due to an inferior lower bound being found, then periods are considered in the same direction, otherwise the direction is reversed. The heuristic is called for the first five levels

of the search tree and every eighth node created thereafter.

6. Standard fixing of non basic variables using reduced costs is carried out. Because of the block dependencies this may lead to the LP solution losing its optimality. In this case we call the LP solver and re-enter the cutting plane generation phase without branching.

7. Many branching rules were tested and the following proved to be the best. The free variables are considered and a subset of these is chosen on the basis of closeness to the value of 0.5. For each variable in the subset we calculate the sum change in the fractional values of all variables dependent on the inclusion and exclusion of the branching variable. Choose the one with the highest minimal sum change in both directions of branching. Strong branching is used if the gap between the lower and upper bound is sufficiently small.

8. If the LP subproblem is not solved within a prescribed maximum time (2 minutes), then the LP optimization is terminated and branching is performed using the rules in 7 above. An attempt is then made to solve the resulting LP's within the specified time. This process is repeated as long as necessary.

9. The cutting plane phase is terminated early if tailing-off is detected or if the LP subproblem is solved optimally in more than a prescribed time (1 minute). Note that adding further cutting planes, even with purging of ineffective constraints, tends to increase the solution times for successive calls to the LP solver.

10. When branching we probe a random subset of variables having the same time index as the branching one. Bounds on variables may also be updated during this process.

11. All our LP subproblems are solved using CPLEX Version 6.0.

4. COMPUTATIONAL RESULTS OF NEW METHOD

Our Branch and Cut algorithm has been implemented in C++ on an SGI Origin 200 dual processor computer. The dual processor was only used to solve the relaxed LP's. The software has been extensively tested both on test data provided to us by our industry partner as well as data from producing mines.

The models in our test data range from 26,208 to 209,664 blocks. In all cases T = 10. We ranged the constraints on the amount of material removed as well as the bounds on the milling requirements so as to cover the large number of cases that can actually occur. For the smaller models solutions guaranteed to be within 0.4% of the optimum were obtained within 12 minutes. For the largest model, solutions guaranteed to be within 2.5% of the optimum were obtained within 4 hours. For these larger models we continue the computations for a further 16 hours and observed there was negligible change in the gap.

Our method generates tight bounds. However, establishing optimality (except, of course, for small problems) is difficult because once we achieve a near optimal solution there are no available cutting planes to remove fractional variables occurring in the same bench level. Note that (6) and (7) give dependencies between variables corresponding to block removal in time and the vertical dimension, but not horizontally.

Following the above extensive testing we applied our method to a producing gold mine. This mine was operating on a schedule generated by MineMax. In order to make a meaningful comparison with this schedule we simulated the same test conditions used by MineMax. This involved reblocking the original block model which contained 23 million blocks to one containing 1363 blocks. However, as reblocking was carried out with different packages, the total value of the undiscounted pit used in our model was 3.3% less. In this application T = 6 and the discount rate was 10%. The constraints involved material movement and an upper bound on the milling capacity (per period). Our software generated 7 good schedules within a total time of 10 minutes. Our best schedule was within 0.27% of the optimum and validated by mining engineers as being realistic. Our schedule yielded an increase of 13.1% in the NPV profit. In fact taking into account the differences in the block model our solution value was at least 15% higher.

The MineMax solution (which was supplied to the mining company by the software author) was obtained through a period by period optimization as the package could not solve globally within the prescribed time limit. An important difference between the two solutions is that ours generates a significantly higher cash flow in the first two periods. This is in fact consistent with the aim of mine planners.

5. CONCLUSION

This paper addresses a fundamental problem in mine planning, that of determining an optimal production schedule. A critical examination reveals considerable deficiencies in the methods currently used by the mining industry. Our new Branch and Cut method resolves these deficiencies and provides powerful tools for improving the efficiency of mining. Computational results on producing mines demonstrate the power of our method.

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